ANALYSIS II BACKPAPER EXAMINATION

Total marks: 100

(1) Let $h : [0,1] \to \mathbb{R}$ be the function defined by $h(x) := 0$ if $x \in [0,1]$ is irrational, $h(0) := 1$, and $h(x) := \frac{1}{n}$ if $x \in [0,1]$ is a rational number of the form $\frac{m}{n}$ where $m, n \in \mathbb{N}$ have no common integer factors except 1. Prove that h is Riemann integrable. (20 marks)

(2) Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$ if $(x,y) \neq (0,0)$, and $f(0,0) = 0$. Is f continuous at all points of $\mathbb{R}^2$? Is f differentiable at all points of $\mathbb{R}^2$? Does f have directional derivatives at $(0,0)$ in every direction? Justify all your answers. (6+6+8=20 marks)

(3) Let $n \geq 1$ be an integer. Consider the three metrics on $\mathbb{R}^n$, the l^1, l^2 and l^∞ metrics. Prove that the topologies on $\mathbb{R}^n$ induced by these three metrics are the same. (20 marks)

(4) Let (X, d) be a compact metric space, and let $f : X \to \mathbb{R}$ be a continuous function. Then prove that f is bounded, and show that f attains its maximum (and minimum) at some point of X. (20 marks)

(5) Let $E \subset \mathbb{R}^n$ be an open subset, and let $f : E \to \mathbb{R}$ be a real valued function such that all the partial derivatives of f are bounded in E. Prove that f is continuous in E. (20 marks)

Date: July 8, 2016.